A Tool for Time-Frequency Analysis of Heart Rate Variability

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Abstract—The analysis of heart rate variability (HRV) signals is an important tool for studying the autonomic nervous system, as it allows the evaluation of the balance between the sympathetic and parasympathetic influences on heart rhythm. Time-frequency analysis of HRV makes it easier to evaluate how this balance varies with time. This work presents a tool for time-frequency analysis of heart rate variability (HRV) developed in Matlab 6.5. Three techniques are available: Short-Time Fourier Transform, Continuous Wavelet Transform, and Time-Variant Auto-Regressive Modeling.

Keywords—AR model, auto-regressive spectrogram, CWT, heart period, HRV, RR, STFT, time-frequency analysis

I. INTRODUCTION

This paper presents a tool for time-frequency analysis of heart rate variability (HRV), which was developed in Matlab 6.5 and incorporated into the ECGLab system [1]. ECGLab is a software for analysis of HRV which was developed at the University of Brasilia-Brasil.

The analysis of HRV is important when studying the autonomic nervous system because it helps in evaluating the equilibrium between the sympathetic and parasympathetic influences on the heart rhythm. The sympathetic branch of the nervous system increases the heart rhythm, resulting in shorter beat-to-beat intervals, and the parasympathetic branch decelerates the heart rhythm, resulting in longer beat-to-beat intervals.

ECGLab, a software developed at the University of Brasilia [1], automatically detects the heart beat instants and creates the heart period signal, based on the beat-to-beat intervals. The statistical analysis module of ECGLab provides indexes that quantify the HRV. The Poincaré analysis module allows the user to evaluate the HRV on isolated heart rates. The spectral analysis module and the sequential trend analysis module help in evaluating the sympathetic-parasympathetic balance on the signal.

However, these classical techniques for analysis of HRV are unable to provide information on how the sympathetic-parasympathetic balance change through time. A tool with that feature could help explaining how the nervous system modulates the heart rate.

In order to improve the ECGLab system in that aspect, a new module was created. The time-frequency analysis module provides the HRV spectrogram, which makes it easy to observe how the sympathetic-parasympathetic activations alternate through time. The spectrogram analysis provides many graphs and indexes that help in evaluating how the sympathetic and parasympathetic influences vary with time.

II. SPECTRAL ANALYSIS OF THE HRV SIGNAL

The spectral analysis of the HRV signal allows one to separate in bands the frequencies related to the sympathetic and parasympathetic activities of the nervous system. The most popular techniques for the spectral analysis of HRV are the Discrete Fourier Transform (DFT) and the autoregressive modeling.

These techniques require the samples of the signal being analyzed to be evenly spaced in time. This is not the case with the HRV signal because its samples are spaced according to the heart beat intervals.

Since the sampling of the HRV signal is nonuniform, in order to use the techniques proposed here, some preprocessing is needed. A solution for this problem is the reconstruction of the signal by interpolation, followed by resampling of the signal at a higher sampling rate [2][3].

Thus, the power spectrum density of the signal is calculated as follows:

- The series of RR intervals is interpolated by cubic splines and the interpolated signal is re-sampled at a higher, uniform rate (usually 2 or 4 Hz);
- The reconstructed signal is multiplied by a 5-minute length window (Hamming and Hanning are the most popular ones);
- The DC component of the signal is removed;
- The spectrum is calculated, using the absolute value of the DFT or the auto-regressive model of the signal.
- The amplitude spectrum is squared, and multiplied by the sampling period.
- When the AR model is used, the result is multiplied by the variance of the prediction error of the model. When the DFT is used, the result is divided by the number of samples in the window.

Fig. 1 shows a comparison between the power spectrum obtained through DFT and its approximation by AR modeling. Although this representation is less accurate than the DFT, many cardiologists prefer to use the AR model because its visualization is simpler, representing the concentrations of sympathetic and parasympathetic energy more clearly. The AR approximation gets closer to the Fourier amplitude spectrum as the order p of the AR model increases. To get a good spectral estimation, the specialized literature recommend values of p=12 for a sampling rate of 2 Hz, and p=15 for a sampling rate of 4 Hz [4].

The power spectrum is divided in 3 bands: VLF (0 to 0.04 Hz), LF (0.04 to 0.15 Hz) and HF (0.15 to 0.5 Hz). Some authors use slightly different divisions, but the important fact here is that the energy contained in the LF band is related to sympathetic activity of the signal, and the...
HF band is related to parasympathetic activity. The power in these bands is calculated based on the area under the curve, and the ratio between them indicates the sympathetic-parasympathetic balance in the segment of signal.

Fig. 1. HRV power spectrum density. The auto-regressive approximation is smoother than the Fourier PSD.

The absolute power contained in the LF and HF bands and the LF/HF ratio are good indicators for detecting alterations in the nervous system behavior. These alterations have been associated with pathological conditions, such as high blood pressure, HIV, Chagas’ disease and ischemic attacks.

In some experiments, the researcher wants to observe the reaction of the nervous system of a subject to specific physical and mental stimuli. In this case, it is usual to obtain the power spectra of two segments of the signal, one before the stimulus, and the other after the stimulus. The two spectra are analyzed separately and the comparison between the obtained indexes for each segment can be used to evaluate the response of the sympathetic and parasympathetic branches of the nervous system to the stimulus. But this technique does not reveal how the components respond as a function of time. This can be accomplished with time-frequency analysis.

III. FOURIER SPECTROGRAM

The Short-time Fourier Transform (STFT) is a classical technique for time-frequency analysis. In this technique, the 5-minute window is replaced by a short-time window (e.g., 30 seconds). This window is shifted sample by sample in time, and for each shift, a new power spectrum is calculated.

Thus, the Fourier spectrogram (Fig. 2) makes it possible to observe the components of the sympathetic and parasympathetic branches of the nervous system as a function of time.

The major shortcoming of the Fourier spectrogram is the tradeoff between spectral and time resolution. For short windows, the time resolution is good, but the spectral resolution is poor. On the other hand, with long windows, the spectral resolution is good, but the time resolution is poor.

Fig. 2. Fourier spectrogram of an HRV signal from a subject with no autonomic dysfunction. The parasympathetic component (HF band) is strong and is present in the whole signal. The sympathetic activity (LF band) decreases in some segments of the signal.

IV. AUTO-REGRESSIVE SPECTROGRAM

An alternative implementation to the Fourier spectrogram is the auto-regressive spectrogram. The proposed technique, called Time-Variant Auto-Regressive Modeling (TVAR), is similar to the technique to obtain the Fourier spectrogram. The main advantage of the technique proposed here is that the pictures generated with the AR model are more straightforward, as illustrated in Fig. 3.

Fig. 3. Auto-regressive spectrogram of an HRV signal from a subject with no autonomic dysfunction. The figure, which was obtained with the TVAR, is clearer that the one obtained with the STFT.

The frequency resolution in the AR spectrogram is affected mostly by the order of the model [5]. That is another advantage of this technique, as one can use a shorter window and thus increase time resolution without losing much frequency resolution. This cannot be accomplished with the Fourier spectrogram.

V. WAVELET SPECTROGRAM

The wavelet transform technique is appropriate for studying non-stationary signals. It represents the time-domain signals over different scales, enabling an identification of both large-scale (low-frequency) and small-scale (high-frequency) fragments. It has been shown that the
The continuous wavelet transform (CWT) helps avoiding problems related to window length and shape, typically encountered with the short-time Fourier Transform (STFT) and Auto-Regressive (AR) spectrum estimation [6].

The wavelet spectrogram, or scalogram, is the squared modulus of the CWT. It is a distribution of the energy of the signal in the time-scale plane, expressed in power per frequency unit (for the HRV signal, ms$^2$/Hz). Both, spectrogram and scalogram can be thought of as smoothed versions of the Wigner-Ville distribution, providing reduced cross-term effects [7].

The continuous wavelet transform of a discrete sequence $x_n$ is defined as the convolution of $x_n$ with a scaled and translated version of the wavelet basis function $\psi_0$:

$$W_n(s) = \sum_{n=0}^{N-1} x_n \psi^* \left[ \frac{(n'-n)\delta}{s} \right],$$

(1)

where the (*) denotes the complex conjugate. By varying the wavelet scale $s$ and translating along the localized time index $n$, one can construct a picture showing how the amplitude of the components in each scale vary with time.

The wavelet basis function for the Derivative of a Gaussian (DOG) function is:

$$\psi_0(\eta) = \frac{(-1)^m e^{\eta}}{\sqrt{\Gamma \left( m + \frac{1}{2} \right)}} d^m (e^{-\eta/2}),$$

(2)

where $m$ is the derivative of the Gaussian. If $m$ equals 2, we have the Marr or Mexican hat wavelet. A real wavelet function like the DOG wavelet returns only a single component and is better suited for isolating peaks or discontinuities. In this work, tests have also been performed with Morlet and Paul basis functions and the results were not satisfactory in terms of definition of the time-scale (spectral) clusters.

A non-orthogonal analysis (such as the one used in this implementation) is highly redundant at large scales, where the wavelet spectrum at adjacent times is highly correlated. The non-orthogonal transform is useful for time series analysis, where smooth, continuous variations in wavelet amplitude are expected. The most noticeable difference is the fine scale structure using the DOG. This is because the DOG is real-valued and captures both the positive and negative oscillations of the time series as separate peaks in wavelet power.

Whereas in the Fourier analysis the frequency resolution $\Delta f$ is constant, in wavelet analysis the ratio $\Delta f/\tau$ is constant. Thus, instead of a linear frequency resolution, a logarithmic resolution is obtained, so that the relative frequency resolution remains the same over the entire frequency interval under observation. In this case, the absolute frequency resolution is obviously much better for lower than for higher frequencies. The importance of logarithmic frequency resolution in HRV may simply be illustrated by looking at the ratios among the frequencies of characteristic peaks – beginning with the peak around 0.013 Hz, each next peak is at about twice the frequency. If a logarithmic frequency axis is used, the peaks are approximately equidistant. The logarithmic frequency resolution of the wavelet analysis makes it possible for this method to capture simultaneously very different rhythms within a single signal.

To make the maps clearer for analysis, we chose to use the inverse of the Fourier period, instead of the wavelet scale, as the dimension of the y axis. For the DOG, the Fourier period is approximately four times larger than the scale [8]. The DOG Wavelet scalogram is included in the time-frequency analysis module of ECGLab and is illustrated in Fig. 4.

![Fig. 4. HRV of the valley of an ST depression ischemic episode and its wavelet spectrogram (scalogram). The trajectories of the VLF, LF and HF time-scale clusters are indicated. The vertical dotted line marks the time instant of the amplitude valley of the episode.](Image)

VI. TIME-FREQUENCY INDEXES

Since the Fourier spectrogram and the AR spectrogram are the combination of the power spectra of short segments of the HRV signal, it is possible to extend the spectral analysis indexes to the time-frequency domain.

Thus, it is possible to obtain curves from the spectrogram which show how the instantaneous absolute power in VLF, LF or HF band varies with time. This can be accomplished by the calculating the power in each band in each PSD, and then plotting these indexes as a function of time. Similarly, it is possible to obtain a curve that shows the LH/HF ratio as a function of time as shown in Fig. 5.

The statistical analysis of these plots (mean, standard deviation, variance coefficient, maximum, minimum, dynamic range, percentiles, etc.) shows information about the sympathetic and parasympathetic control over the heart rate.

From the variation of LF/HF ratio curve, it is possible to extract another index, the ratio of areas. This index measures the global sympathetic-parasympathetic equilibrium in the
signal. In Fig. 5, the dotted line is an equilibrium threshold, which indicates where the LF/HF ratio equals 1. Above this line, the curve reveals sympathetic dominance. Below the threshold, the parasympathetic influence is dominant. The index is obtained by calculating the ratio between the areas above and below this line as shown in Fig. 6.

![Fig. 5. Time-frequency plots obtained from the spectrogram: variation of the instantaneous absolute power in the VLF, LF and HF bands. The LF/HF ratio variation curve is also shown.](image)

![Fig. 6. The ratio of areas is calculated as the ratio of the areas above and below the equilibrium line.](image)

VII. TIME-FREQUENCY ANALYSIS MODULE

In order to make the time-frequency analysis easier to use for physicians and researchers, a time-frequency module was added to the ECGLab system. This module implements the techniques presented in this work with a simple and easy to use interface (Fig. 7). The parameters to obtain the spectrogram are easy to input and the time-frequency indexes are obtained clicking the mouse.

![Fig. 7. A view of the time-frequency analysis module. In this window, the HRV signal is being analyzed with the AR spectrogram, and the LF/HF ratio variation curve is being displayed.](image)

VIII. CONCLUSION

The software presented in this work was developed in order to make the time-frequency analysis easy to be used by researchers. As a general evaluation of the obtained results, it can be deduced that the study of HRV in the time-scale domain is a technique of clear clinical interest in the monitoring of myocardial ischemia, high blood pressure, Chagas’ disease and others, due to its detailed display of the dynamics of the time-scale clusters and its adequate representation of non-stationary signals over different scales, enabling an identification of both large-scale (low-frequency) and small-scale (high-frequency) fragments.

REFERENCES

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Erratum

The correct sequence of steps for the calculation of the power spectrum density of the HRV signal is:

• The series of RR intervals is interpolated by cubic splines and the interpolated signal is re-sampled at a higher, uniform rate (usually 2 or 4 Hz);
• The DC component of the interpolated signal is removed;
• The resultant signal is multiplied by a 5-minute length window (Hamming and Hanning are the most popular ones);
• The spectrum is calculated, using the absolute value of the DFT or the auto-regressive model of the signal.
• The amplitude spectrum is squared, and multiplied by the sampling period.
• When the AR model is used, the result is multiplied by the variance of the prediction error of the model. When the DFT is used, the result is divided by the number of samples in the window.

The order of steps 2 and 3 were mistakenly switched in page 1 of the paper. The DC component should be removed before windowing, not after.